# national accelerator laboratory

Calculated Beam Properties for the NAL 750 keV Transport System

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### INTRODUCTION

The proposed pre-linac beam transport system has been tested using computer programs at Brookhaven National Laboratory. The results indicate that only small modifications will be needed to provide the approximate beam shape desired. Further computer runs will be needed to obtain an optimized beam.

### I. PURPOSE OF TRIP

The primary reason for a trip to BNL was to give the proposed pre-linac transport system of C.D. Curtis a simulated test on computer programs now in operation at BNL. This was deemed necessary because of the significance of space charge forces in 750 KeV beams of the order of and larger than approximately 100 mA. With a proposed current of 225 mA, containment of the diverging beam from the pre-injector with acceptable beam-emittance growth is a problem. In particular, a calculation to simulate such a beam without a rather elaborate computer program is impractical if not impossible. To insure having an operational design the BNL program was to be used at least to the extent of demonstrating a transport system which would give the approximate shape desired.

### II. DESCRIPTION OF THE BNL PROGRAM

The BNL computer program for a transport system is a modification by R. Chasman of programs developed by her for simulating beams in linacs. The basic program for the linac is

<sup>1.</sup> R. Chasman BNL Internal Report AGS-CD-31

similar to the PARMILA program of LASL developed by D. Swenson and others.

The transport program modification actually resulted in two separate programs. One program treated a continuous beam and thus had only transverse coordinates. It was called the DC version. The other program was called the Buncher version; it included the longitudinal coordinate and required considerably more computer time. As indicated by the name, the Buncher version included the possibility of putting in, as an impulse, the energy change as a function of particle phase due to a buncher cavity. Also an artificial bunching effect could be incorporated in the DC version by increasing the effective beam current in calculation elements as the beam was traced through the transport system.

Quadrupoles were treated as ideal thick lenses in the matrix approximation. An ideal region of constant gradient is assumed for the quadrupole with appropriate effective length and sharp cutoffs. To add the fringe field effects of a more realistic quadrupole, impulse corrections were added to the particle coordinates at the entrance and exit of the ideal quadrupole region. These corrections were those needed to satisfy Maxwell's equations at the edges.

The important effect of the space charge forces was treated in two dimensions in the DC version and in three dimensions in the Buncher version. The difference in the computer time requirements of the two versions comes primarily from the space charge calculations. The transport system was divided into elements of either quadrupoles or drift spaces. At one point in each element the space charge subroutine calculated impulses to the coordinates to represent the effect of space charge forces over the length of the element under consideration. In the calculation the beam is represented by a number of particles (less than 500). Each particle is treated as a source sphere of charge (infinite cylinder in the DC version) representing its fraction of the total charge in the beam. The volume around each

particle is estimated from the root-mean square beam dimensions such that it would enclose 8 particles on the average. Then the number of particles contained around each particle in a volume with twice the radius of the estimated source volume radius is calculated. Each particle's source volume radius is then modified according to the actual density. The DC version did not have this density compensation. The force on a particle from a source volume of charge is calculated by the normal law if the particle is outside the source volume. If it is inside, the force is reduced by considering the charge of the source particle to be spread uniformly in the source volume.

Beam size was estimated by RMS coordinates calculated by the usual formula:  $\sqrt{2}$ 

 $\chi_{rms} = \sqrt{\frac{\leq \chi_i^2}{N}}$ 

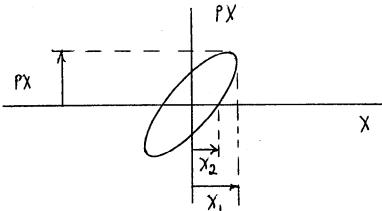
where N is the number of particles. Since the beam growth in terms of emittance area is an important parameter, an RMS area was calculated as originally suggested by R.L. Gluckstern. The RMS area is defined by the equation:

$$A_{rms} = \sqrt{2 \chi_i^2 2 \gamma_i^2 - (2 \chi_i \gamma_i)^2} / N$$

One notes that without the XY term this is just the product of the RMS dimensions and is the RMS area of all the particles folded into one quadrant. The XY term allows for nonsymmetries about the axes.

The area calculation deleted up to 10% of the particles at each end of a distribution if the particles were separated too much from the bunch. Only the relative growth of the areas was intended to be a useful number.

Both versions of the program were set up to randomly populate the ellipses inferred from three initial parameters for each transverse axis. A linear uniform distribution was used for all but the PX and PY coordinates in the DC version in which a quadratic distribution was used. The following diagram defines the parameters:

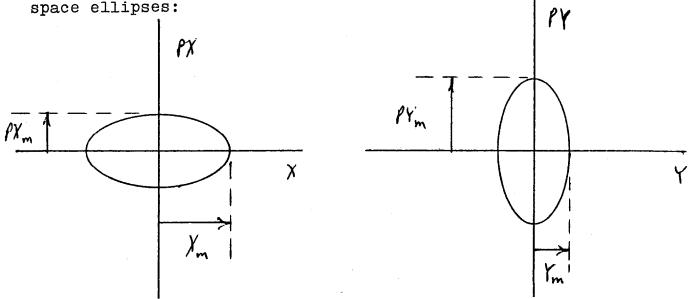


PX is  $\beta \chi \chi'$ . All quantities are in units of meters. Note that the population is not independent in the X and Y planes because, for example, a particle with the maximum value of X cannot have at the same time the maximum value in Y (i.e. a rectangular shape in the X-Y plane). The Buncher version also uniformly populated the particle phase coordinates. An energy spread option was available but was not used.

# III. PROPOSED TRANSPORT SYSTEM

The system proposed by C.D. Curtis consisted of three quadrupole triplet sets. The beam is to come to a waist between the first and second triplet in the region of a mass analyzer and between the second and third triplet in the region of a buncher cavity.

The beam shape needed for the linac is shown by the two phase



where X is the focusing axis of the first linac quadrupole. The ratios Xm/PXm and Xm / Ym are determined from properties of the quasi-periodic quadrupole system of the linac. Other conditions come from the beam emittance, linac acceptance, and the relative size of the beam to the drift tube bore.

The first two triplets consisted of 12 cm, 24 cm, and 12 cm quadrupoles with 3 cm gaps between them and a 7.5 cm diameter bore. The third triplet was selected with 8-3-16-3-8 cm lengths. The most immediate concern was containment of the diverging beam from the pre-injector by the first triplet. Since a quadrupole has one focusing and one defocusing axis, the already diverging beam of the pre-injector must undergo even more defocusing in one axis before it will see any focusing forces. This makes it difficult to obtain a small beam size in that axis. The maximum size of the beam and the quadrupole length have been related to the emittance growth of the beam due to fringe field effects by R.L. Gluckstern. Consequently, one wants to keep the beam size small and use long weak quadrupoles to minimize the emittance growth. Small beam size is not consistent with long weak quadrupoles in the first triplet of the system.

The beam coming from the preinjector was taken as a circular diverging beam with the following parameters:

X1 = Y1 = .0156m

PX = PY = .00191m

X2 = Y2 = .0014m

I=225 mA

 $E = PX \cdot X2 = 4.45 \text{ mr cm}$ 

These values represent the projected operational parameters of the preinjector. For the Buncher version the cavity voltage was taken as 30 kV. The transverse beam parameters at the buncher (the beginning of buncher program) were picked to

<sup>3.</sup> R.L. Gluckstern, NAL Report FN 166

agree roughly with the size and shape of the beam at the buncher position in the DC program. The values were:

X1 = .00826 m

PX = .00053 m

X2 = .0081 m

Y1=.00786 m

PY=.00055 m

Y2=.00753 m

 $E = PX \cdot X2 = 7.13 \text{ mr cm}$ 

## IV RESULTS AND THEIR APPARENT SIGNIFICANCE

The DC version was run with drift spaces divided into lengths nominally about 6 cm. The bunching effect available by synthetically increasing the current was used only on several of the later computer runs. The Buncher version was run only from the front of the buncher cavity to the linac, and even so consumed significantly more computer time.

The first eight runs were made with a maximum divergence X' of about 50 mr. instead of the desired 32 mr. They are useful even so to show the difficulties encountered if the preinjector cannot be made to give the smaller divergence angle. The problems seen were of containment not dissimilar to the effect of the 3 cm first triplet spacing to be explained shortly. Further discussion will concern only the runs made with the projected divergence.

Computer Run 9 was made using the first triplet of the proposed BNL transport system. The values are:

Triplet Spacing cm	Gradient Gauss/cm
10-2-20-2-10	600-500-600
12-3-24-3-12	485-392-485
8-3-16-3-8	1255–1020–1255

The five numbers under the column labeled triplet spacing are, the lengths of individual quadrupoles and drift spaces between them. Except after the third triplet which was too

strong, the beam was reasonably well contained and had waists close to the desired positions. Figure 1 shows X and PX coordinates up and Y and PY coordinates down.

Figure 2 for Run 10 shows the effect of increasing the space between the quadrupoles of the first triplet to 3 cm. The waists are smaller and slightly shifted; the maxima are larger to the point of beam loss in the third triplet. This behavior was typical of that obtained in the earlier runs with 50 mr. divergence. A beam shape approaching that desired was not obtained in those eight tries at the larger divergence. It appears that for such a divergence shorter quadrupole triplets such as 8-16-8 or shorter would have to be used.

Figure 3 shows Run 11 which had a first triplet of: 12-2-24-2-12 cm at 485-392-485 g/cm.

The maximum size in Y occurring in the first triplet can be seen to be larger than was obtained in Figure 1 with a 10-2-20-2-10 triplet. Run 12 had an even shorter triplet of 8-2-16-2-8. The maximum in Y in the first triplet was smaller yet, but the gradients used were incorrect because the Y motion did not decrease to a desirable waist after the first triplet. As a result the Y motion became so large in front of the second triplet that particle loss occurred.

Run 13 shown in Figure 4 is Run 11 with the sign reversed in the second triplet. That is, Run 13 had +-+, +-+, and +-+ as opposed to +-+, -+-, and +-+ of Run 11. The second waist in X is not correct but could possibly be adjusted.

Run 14 had a first triplet of 12-3-24-3-12 cm and thus bears the relation to Run 11 that Run 10 does to 9; that is, the spacing between the quadrupoles of the first triplet were increased from 2 cm to 3 cm. The results were similar with larger Y motion and eventual particle loss. The third triplet values were changed to 900-800-900 G/cm for reasons that will be apparent from the Buncher runs.

Run 15 was Run 11 with the gradients changed in the first triplet by +3%/+1.5%/+3% and in the second by 0%/-2%/0%. The

third triplet was run at values of 945-840-945 G/cm. These changes produced a slight improvement in X at the first waist and in Y at the second waist, but an undesirable increase in the second waist in X. More runs will be needed to optimize the beam shape and size.

Figure 5 shows Run 16 which had the gradients of Run 15 but a reduced input beam size. The maximum coordinates of the ellipse were decreased roughly by a third. However, the crossing on the length coordinate was not changed. This means the ellipse was a third shorter but of the same width. The area or emittance thus was also decreased by about a third.

X1=Y1=.01 m PX=PY=.0012 m X2=Y2=.0014 m E=PX·X2=2.8 mr cm

PX and PY correspond to a divergence of about 20 mr.

Figures 7 and 8 show Buncher Runs 3 and 5. Run 2 (not shown) indicated that the first gradients proposed (1255-1020-1255 G/cm) were too high. Run 3 used gradients of 900-800-900 with much better results. Run 4 (not shown) had adjusted gradients of 945-840-945 that gave slight decrease in X and Y coordinates at the linac, but at the expense of an increase in PY. Note that the shape and relative values of the coordinates for Runs 3 to 5 are roughly what is needed at the linac. The absolute beam size is too large however. Run 5 in Figure 8 shows the effect of changing the quadrupole spacing from 3 to 2 cm. The net effect is a decrease in PY but an increase in X and Y back to the values of Run 3. The last run, number 6 (not shown) used a 9-2-18-2-9 triplet that gave a beam loss in the middle of the triplet with the gradients used. In changing from one length quadrupole to another the gradients were changed such as to keep the lens focal length the same in the thin lens approximation. It can be seen from the times this was tried in both sets

of computer runs that the results desired were not always obtained by this procedure. Space charge forces with thick lenses prevent an easy prediction of beam behavior.

Table 1 shows the RMS areas for Runs 9 to 16 calculated by the program for both the Y-PY and X-PX emittances at the beginning of the transport system, about 15 cm after the first triplet, about 15 cm before the second triplet, and at the buncher cavity location between the second and third triplet. The last row shows the number of the triplet near the location of particle losses.

Table 2 shows the areas from the buncher program at the buncher cavity and at the entrance to the linac or end of the transport system. The row labeled Particle Number gives the number of particles simulating the beam. The rows labeled KB and KBB give the number of beam particles that occurred either between phases of  $\pm \varphi_{\varsigma}$  or  $+\varphi_{\varsigma}$  and  $-2 \varphi_{\varsigma}$  of the linac. These numbers give an estimate of bunching efficiency.

Table 3 gives the ratios of the areas or effective area growth in X and Y across the first triplet, across the first beam waist, across the second triplet, and the total from beginning of the transport system to the buncher. The last four rows give the products of the X and Y growths for the same regions. These numbers are related to dilution of the beam brightness.

Table 4 gives the growth from the buncher cavity to the linac from computer runs of the Buncher version.

Looking first at the end result in the last row of Table 3, the run numbers in apparent order of increasing beam growth are 16, 15, 13, 11, 9, 12, 10, and 14. Run 16 had a smaller input beam than the others so that it is interesting to the extent of showing the effect of that decrease in beam input. The next three runs were all run with 12-2-24-2-12 cm first triplets. Next in the list come the 10-2-20-2-10 first triplet run (number 9) and 8-2-16-2-8 first triplet run (number 12). The last two in the list show the bad effect of having 3 cm

between quadrupoles rather than 2. The first conclusion from the order of the runs is that the longer quadrupole triplets cause less beam growth than shorter ones. However, looking at Runs 13 and 11, Run 13 had only one difference in operating conditions from Run 11. That difference is a change in the sign of the gradients in the second triplet. If a plus sign means focusing in the X direction and defocusing in the Y, the quadrupoles have the following sign:

This change in sign means that in Run 13 the maximum beam size in the second triplet occurs in Y as it does in the first triplet. In the other runs the maximum beam size occurs in Y in the first triplet but in X in the second triplet. There thus appears to be some 'cancellation" of beam growth effects by having the maxima occur in the same coordinate. Some type of cancellation effects through several quadrupoles was suggested by R. Gluckstern<sup>3</sup>.

In comparing Runs 11 and 15 there appears to be benefit gained in adjusting the gradients. In fact, the changes between these two runs were 3% or less. R. Gluckstern also suggested possible reductions of beam growth by small gradient adjustments. These adjustments were tried only in an attempt to move the waists to a more desirable position. The result did in fact show an improvement in X at the first waist and in Y at the second waist. However, the second waist in X became slightly larger. Now taking time to look at rows other than the last in Table 3, the first thing one notices is that there are regions of decreases in area. In particular, Run 11 as compared to Run 9 shows that a greater increase in area is observed across the longer quadrupoles, but a greater decrease in area is seen across the waist after the longer quadrupole. The observed decreases and part of the increases must be attributable to a density effect in the definition of RMS area used. It is difficult to interpret the area or area growths given.in terms of an

acceptance ellipse for example. Figures 9 through 19 show some computer printouts of either the Y-PY or X-PX planes from some selected positions in the transport system of Runs 10 to 15. Each run started out by populating an ellipse in the X-PX and Y-PY planes using a pseudo-random number generator with a flat distribution. A linear uniform distribution was used in X and Y, but in PX and PY the distribution was taken from a squared random number. If R is the random number selected in a flat distribution between zero and one, then the coordinates were selected according to the following equations:

$$X=X_{m}$$
 (2R-1)  
PX=PX<sub>m</sub> (R<sup>2</sup>-1)

This gives X a flat distribution between +Xm and -Xm, but PX is chosen between -PXm and zero with the density increasing towards -PXm. Symmetry is forced on the other particles selected to fill the four dimensional volume by using the same values of X, PX, Y and PY and selecting three other particles by changing the signs:

1	2	3	4
X	<b>-</b> X	X	<b>-</b> X
PX	-PX	PX	-PX
Y	Y	<b>-</b> Y	<b>-</b> .Y
PY	PY	-PY	-PY

Figures 9 through 19 are obviously not ellipses.

It appears that the effective area growth pertinent for the acceptance of the linac might be considerably larger for a distribution like Figure 16, for example, than the area of the particles.

To add further confusion to the real meaning of the area and its growth, consider the first two rows of Table 1. Consider these areas only relative to each other so that the absolute scale is of no importance. These areas are the RMS areas calculated by the program of the initial particle distribution.

In runs 9 to 15 the initial values given to the three ellipse parameters are identical. The pseudo-random number generator is given the same starting number and will therefore generate the same sequence of numbers. In Runs 11 to 15 the ellipses were populated with the same number of particles (348). Y stays the same and X stays the same. The 4% difference between X and Y must be just due to the different population of the same ellipses (ellipse parameters are identical). The number of particles was different for Runs 9 and 10 (112 and 448 respectively). It is not clear why Run 9 with 112 particles should show only a 4% spread between the Y and Y planes while Run 10 with 448 shows a 22% spread between them. In Run 16 the maximum dimensions of the ellipse were about two thirds of the values for the earlier runs. However, the width of the ellipse was the same. Had all three of the ellipse parameters been reduced by one third, one would expect an identical distribution of particles because the relative scaling factor has no effect. However, with an effectively wider ellipse, particles that were just outside before are moved inside. One expects the average area to be reduced by two thirds for Run 16, and the average of the two plane areas was reduced approximately by that factor. A 35% spread about the average of the X and Y areas is difficult to explain.

In light of the previous discussion about the areas calculated, it is not too clear how seriously one should take the observed 10% area growth difference between the 12-24-12 cm triplet of Run 11 as compared to the 10-20-10 triplet of Run 9. The choice will have to come from other considerations with a detailed look at the beam shape development. The rough total growth through the whole transport system can be obtained from the product of the growth in DC Run 11 times that of Buncher Run 5, for example. One obtains a number of 3.7 which is not a very pleasing one. It must be said, however, that in view of the selection of the initial distribution a diverging beam ellipse would have a slight hollow effect in the divergence coordinates that would be reflected slightly in the transverse coordinates. A real beam would more likely have a higher density in the center rather than at the edges of the ellipse. Therefore,

it might be expected that the area growth calculated emphasizes the edge particles, thus giving an increased magnetic edge effect and area growth. Certainly the density effect in calculating areas discussed above must also be emphasized by this particle distribution.

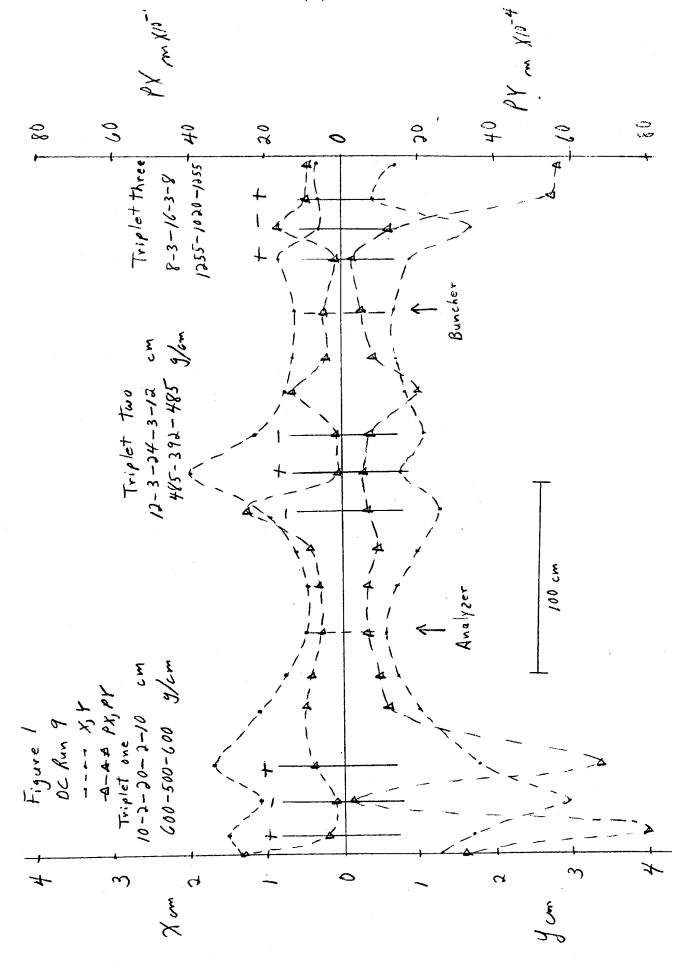
Figure 20 shows a Y-PY plot just in front of the linac for Buncher Run 5. Figure 21 shows the phase energy plot at the same point. At the beginning of the run this plot was a straight line on the phase axis (DC beam).

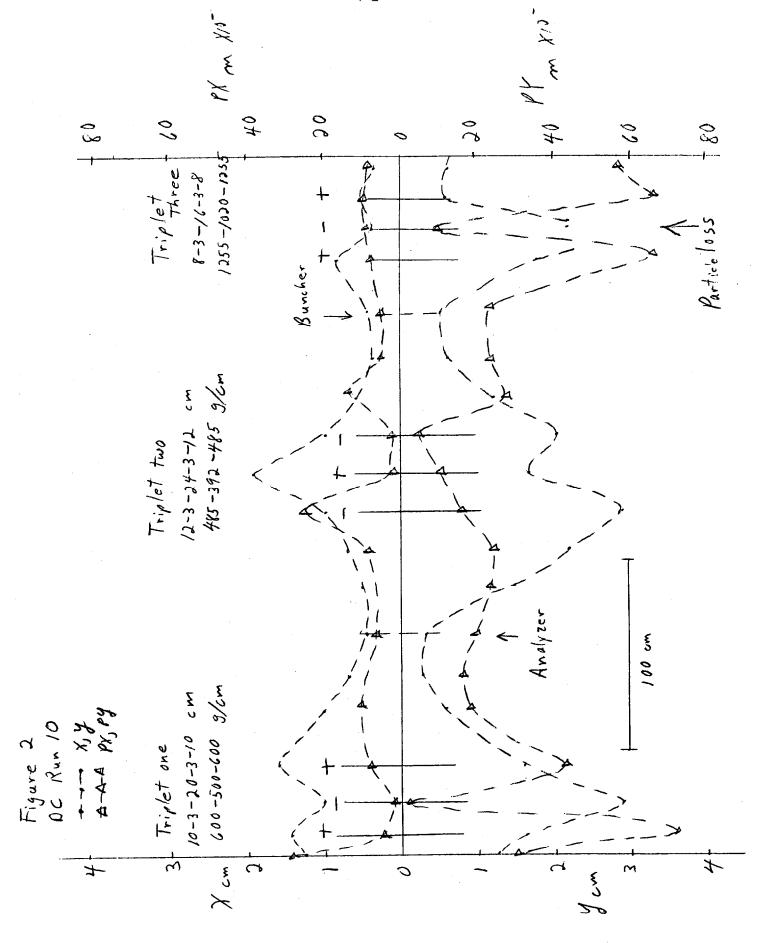
### V. CONCLUSION

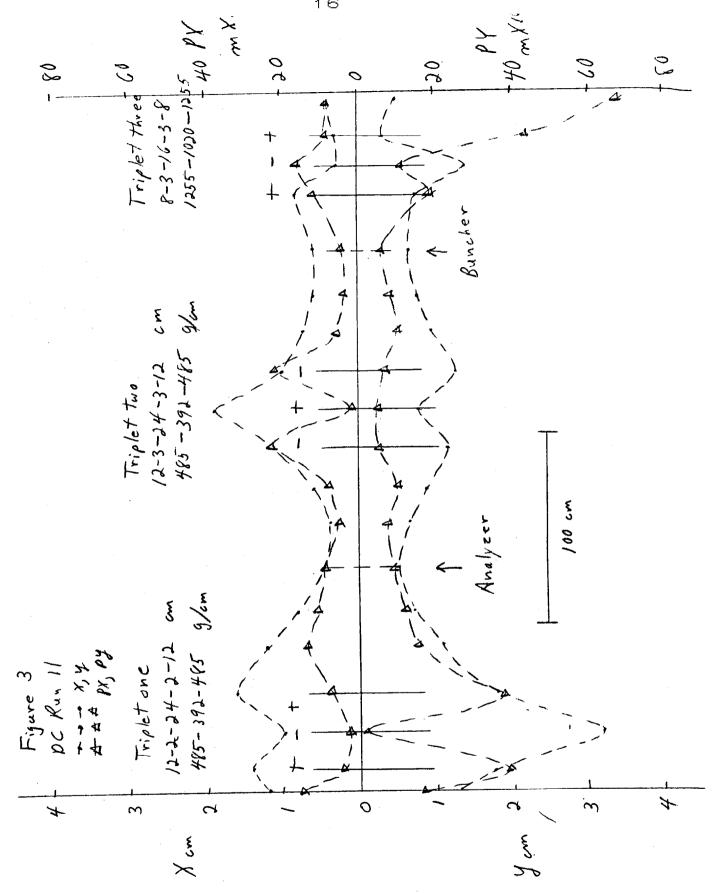
One point is clear if one believes the program results at all, and that is the need to go to closer quadrupole spacing in the first triplet. The choice between different length quadrupoles in the triplet is a trade between slightly larger beam size and slightly smaller beam growth. The difficulty of determining the meaning of the RMS beam dimensions and areas prevents an easy or even clear choice.

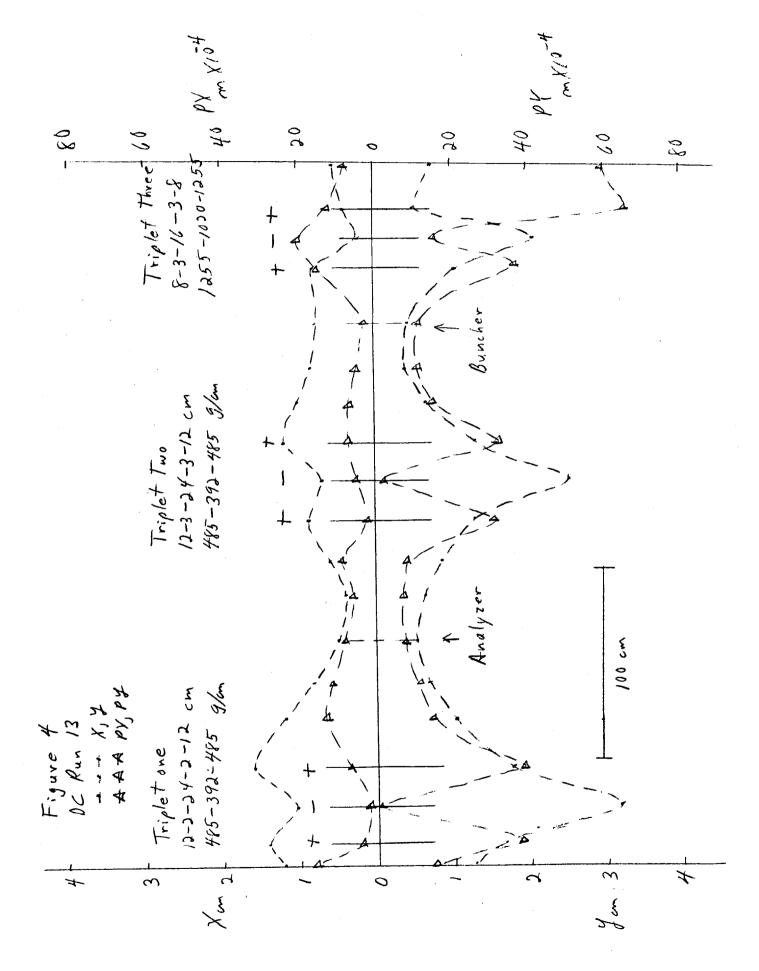
On the plus side, the programs have demonstrated the appropriate beam shape and seem to indicate that the desired beam can be obtained by some adjustment of the triplet strengths and possibly the position of the third triplet. This in itself was the purpose of the trip to BNL to show up any great difficulties with the proposed beam transport design. Construction can proceed with less fear of operational difficulties later.

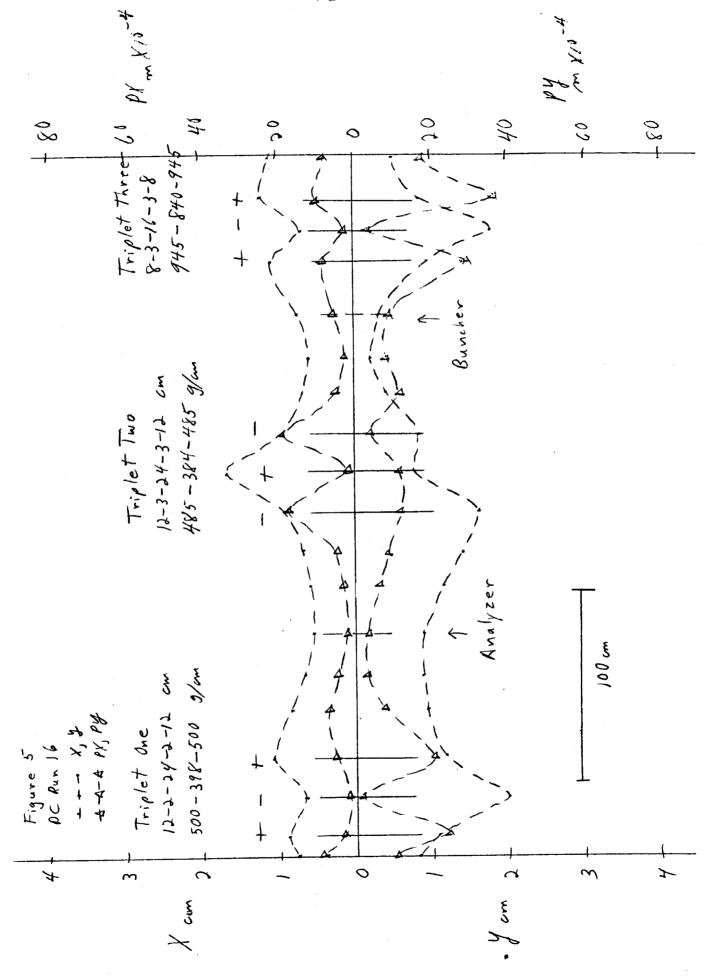
However, it is also clear that more study of the quadrupole gradients and positions is desirable. It will be necessary at some point in time to find the gradients necessary to provide a beam at the linac very close to the matched condition.

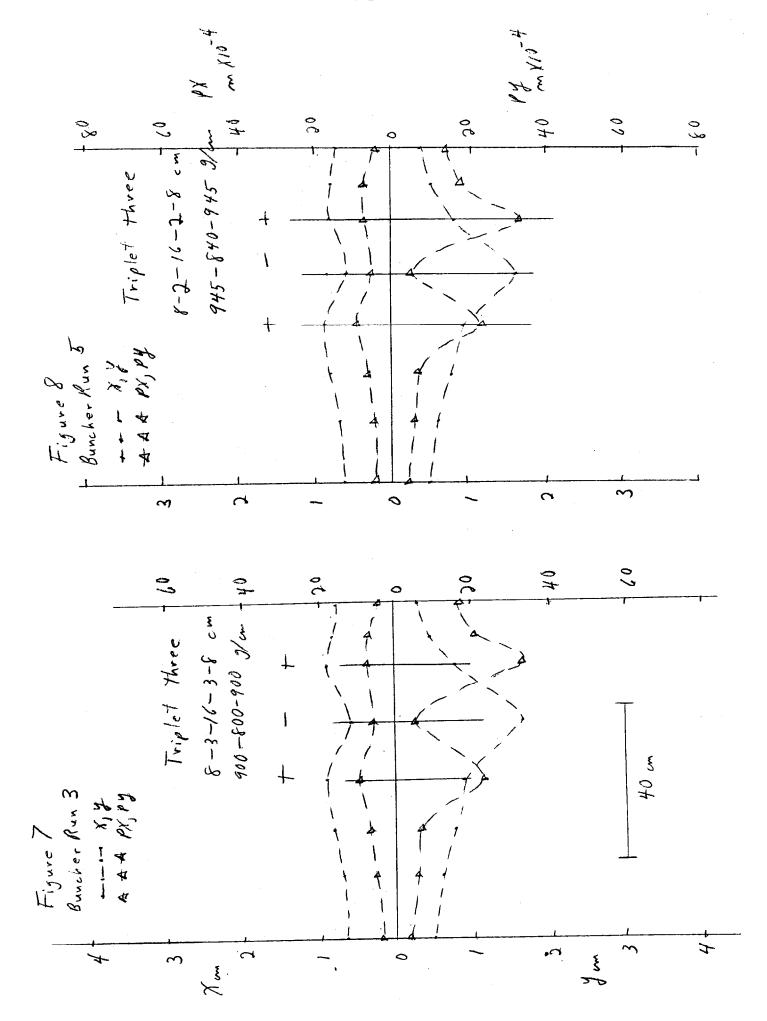




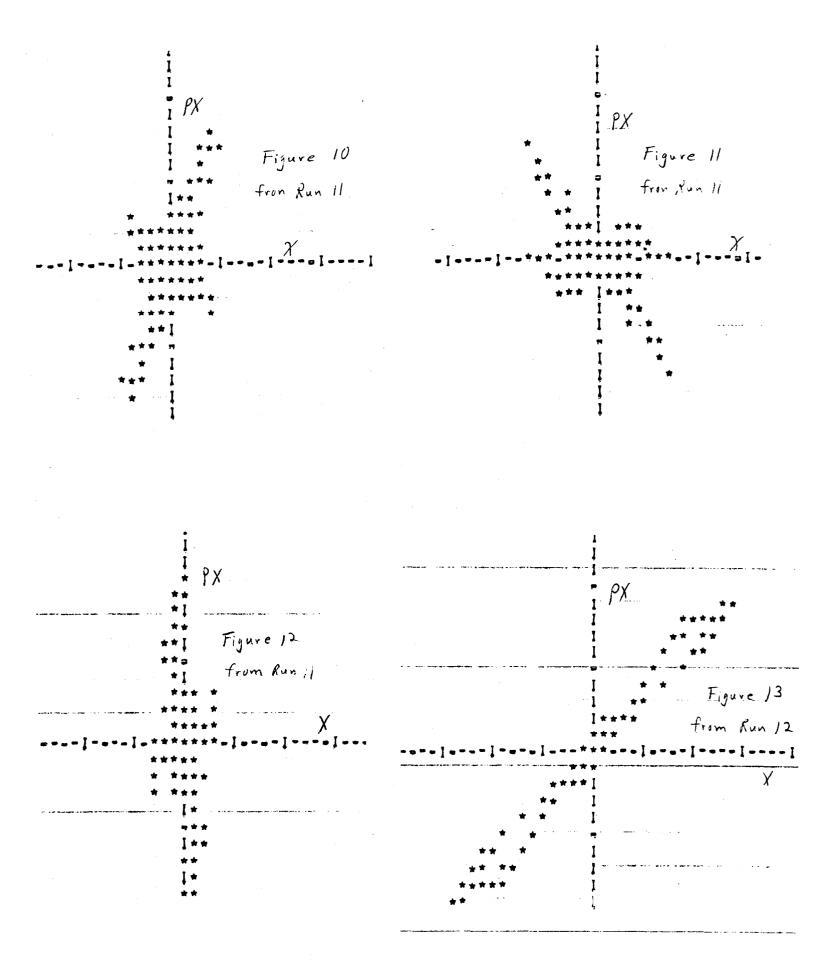








I PX Figure 9 from Run 10



	PX	PX
Figure 14 from Run 12	1 ****** 1 ****** 1 ******	Figure 15 from Run 13
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	PX	
	Figure 16	Figure 17. from Run 15
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Table I RMS Areas times  $10^{-7}$  DC Runs

Run N	<del> </del>	9	10	11	12	13	14	15	16_
Y		4.18	3.68	3.95	3.95	3.95	3.95	3.95	3.32
In X			4.56	4.11	4.11	4.11	4.11	4.11	2.33
Dofore voice	Y	8.49	8.61	10.36	9.25	10.36	10.5	10.6	4.28
Before wais	X	6.37	6.37	5.99	6.73	5.99	5.80	5.91	2.58
After waist	Y	7.54	8.44	8.00	10.2	8.00	14.7	8.18	4.01 <sub>N</sub>
Arter warst	X	6.17	5.51	5.13	5.59	5.13	5.18	4.90	2.42
Buncher		6.49	15.9	6.44		7 <b>.</b> 15	33.1	6.05	3.14
buildier		·	5.06	6.08		5.04	5.14	5.61	3.42
Loss Triple	t		3	With reviewing	2,3	-	3		

Table II RMS Areas times  $10^{-7}$  Buncher Runs

5	5.14	7.63	797	225	247
4	5.14	7.57	397	225	247
2	5.14	7.63	397	226	248
2	5.14	10.3	397	220	244
Run N	Y Buncher X	Y Linac X	Part,	KB	KKB

Area Growth DC Version

	0	10	11	12	13	41	15	9 7
Y triplet	2.03	2.34	2.63	2.34	2.62	2.66	2.68	1.29
×	1.69	1.40	1.46	1.64	1.46	1.41	1.44	1.11
<b>≯</b> 1	.89	. 98	.77	1.10	17.	1.4	.77	.94
first waist X	.91	98 .	98.	.83	.86	.89	.83	.94
Ħ	98.	1.88	. 81		. 89	2.25	.74	.78
Second triplet X	1.13	. 92	1.19		86.	66.	1.14	1.41
	1.55	4.32	1.63	*2.58	1.81	8.38	1.53	29 <b>36.</b>
	1.73	1.11	1.48	*1.36	1.23	1.25	1.36	1.47
First triplet	3.43	3,28	3.84	3.84	3.83	3.75	3.86	1.43
First waist	.8	.84	99.	.91	99.	1.25	.64	88.
Second triplet	.97	1.73	96.		.87	2.23	.84	1.10
	2.68	4.80	2.41	*3.51	2.22	10.48	2.08	1.40

\* Growth only up to second triplet

Table IV

Area Growth Buncher Version

Run #	2	3	4	5	
Y	2.00	1.48	1.47	1.48	
X	1.10	1.03	1.06	1.04	
Product	2.2	1.52	1.56	1.54	

## ACKNOWLEDGEMENTS

I certainly wish to express my gratitude to the Brookhaven staff for the help received and for making our stay there a pleasant one. Particular recognition must be given to Rene Chasman for the use of the programs developed by her and for the time she spent explaining and helping to run the programs.

Acknowledgement must also be made of the many discussions about the whole area of transport system, results, meaning, etc. to C.D. Curtis.